2 Question 1 - Water Flowing out of a Conical Tank

Water flows out of a small circular hole at the bottom of an inverted conical tank in which the radius r increases with elevation (similar to an ice cream cone). There is no inflow to the tank - only an outflow. The volume of water in the tank is to be approximated by:

$$\forall = \frac{1}{3}\pi r^2 h$$

where... h = depth of water above the bottom of the tank i.e. above the pointed end of the tank

r = radius of the tank at elevation h.

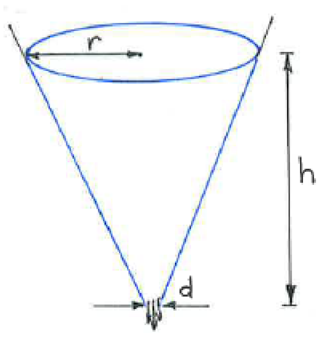

Figure 1: Flow from a circular hole with diameter d at the base of a conical tank

Initially, the depth of water is $h_o = 4.0\,m$, and the corresponding radius at the top of the water surface is $r_o = 3.0\,m$. The radius of the circular hole at the bottom of the tank is $d = 5\,cm$. Estimate the following:

1. Find an expression which enables you to determine how long it takes for the water level in the tank to fall from one level h_o at time $t = 0$ to level h at time t? Then determine how long it will it take for the water level to drop from an initial depth of $h_o = 4.0\,m$ to a depth of $h = 2.0\,m$?
2. When the water depth in the tank is $2.0\,m$, what is the (vertical) rate of fall of the water level? Give your answer in units of (mm/s).
3. How long will it take for the tank to empty when the initial water depth is $h_o = 4.0\,m$?
4. Very briefly state under what physical circumstance do you think that this calculation will be less accurate, and why?

3 Question 2 - Water Flowing from a Tap

Water flows vertically from a tap in a coherent stream. The outlet diameter of the tap is D and the initial vertical velocity as the water flows out of the tap is v_1. If the stream of water leaves the tap and falls through a vertical elevation drop of magnitude h, just before it reaches the ground, find an expression for the diameter (d) of the stream just before it reaches the ground, in terms of the acceleration due to gravity g and the variables mentioned in this question?

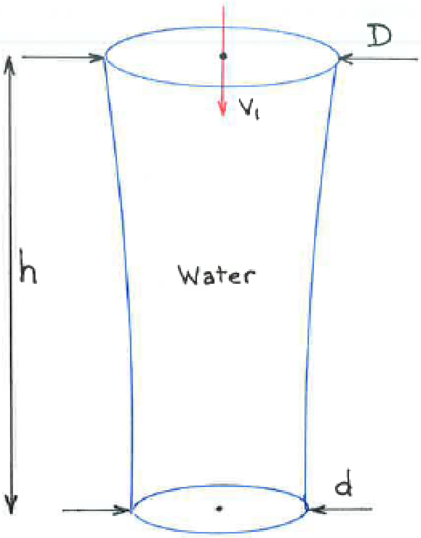

Figure 2: Water being discharged from a tap with diameter D undergoes free fall.

4 Question 3 - Average Flow Velocity and Momentum Fluxes in an Open Channel Flow

Water flows in a wide open channel at a depth h. In this problem, you need only consider a unit width of flow i.e. $B = 1\,m$ wide. The flow velocity is not constant through the water column (i.e. through the depth), but varies according to the commonly used approximation for a (turbulent) flow known as the one seventh power law (see Figure 3), as:

$$\frac{u}{u_s} = \left(\frac{z}{h}\right)^{1/7}$$

where... u_s = flow velocity at the water surface (m/s)

u = flow velocity at elevation z above the bed (m/s)

z = elevation above the bed of the channel (m)

h = depth of flow in the channel (m)

(You will become more familiar with the power law when you cover the later topic of boundary layers, but for the present problem, the equation above suffices.)

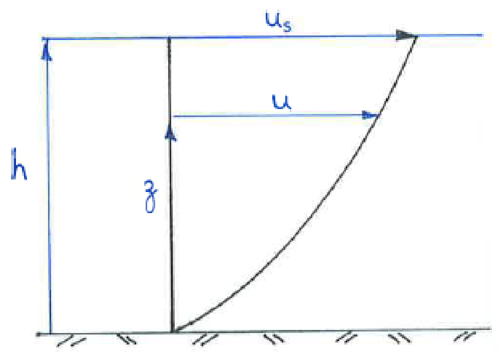

Figure 3: One seventh power law for the velocity profile with depth $\frac{u}{u_s} = (\frac{z}{h})^{1/7}$.

Complete the following:

1. Find an expression for the depth-averaged flow velocity $(\bar{u})$ in terms of the surface velocity u_s where $Q = A\bar{u} = \int_0^h u.dA$ and $dA = B.dz = (1)dz$ and $A = (1)h = h$.

2. Find an expression for the momentum flux $(\dot{M})$ past a cross-section $1\,m$ wide, in terms of ρ, h, $\bar{u}$. In other words find the value of the constant k in $\dot{M} = k\rho h\bar{u}^2$.

Hint: ***Some*** of the parameters in the following hierarchy of ***fluxes*** may be useful:

$$\dot{m} = \int \rho.dQ \qquad \textbf{(mass flux)}$$
$$= \int \rho(u.dA)$$

(provided ρ, u are constant over the cross-section,
or, provided ρ is constant over the cross-section **and**
$u = \bar{u}$ = average velocity over the cross-section)

$$= \rho.Q$$

$$Q = \int dQ \qquad \textbf{(volume flux)}$$
$$= \int u.dA$$

(provided u is constant over the cross-section,
or, provided $u = \bar{u}$ = average velocity over the cross-section)

$$= u.A$$

$$\dot{M} = \int \rho.dQ.u \qquad \textbf{(momentum flux)}$$
$$= \int \rho(u.dA)u$$
$$= \rho.Q.u$$

(provided ρ, u are constant over the cross-section)

$$\dot{E} = \int \underbrace{\rho g}_{\gamma} H.dQ \qquad \textbf{(energy flux)}$$
$$= \int \rho g H.(u.dA)$$
$$= \gamma Q.H$$

(provided ρ, u, H are constant over the cross-section)

5 Question 4: A Slot Jet Entering an Open Channel with a Sluice Gate

A slot jet with thickness t and width B enters an open channel with a rectangular cross-section also of width B. The slot jet is inclined downwards at an angle θ to the horizontal. The channel is closed by a wall at the left hand side, and on the right hand side, there is a sluice gate which allows the water to escape under it. Three sections are defined (see Figure 4):

- Section ① is close to the left hand closed end of the channel,
- Section ② is upstream of the sluice gate where the flow in the channel is parallel to the channel bottom, and
- Section ③ is downstream of the sluice gate where the flow is again parallel to the channel bottom.

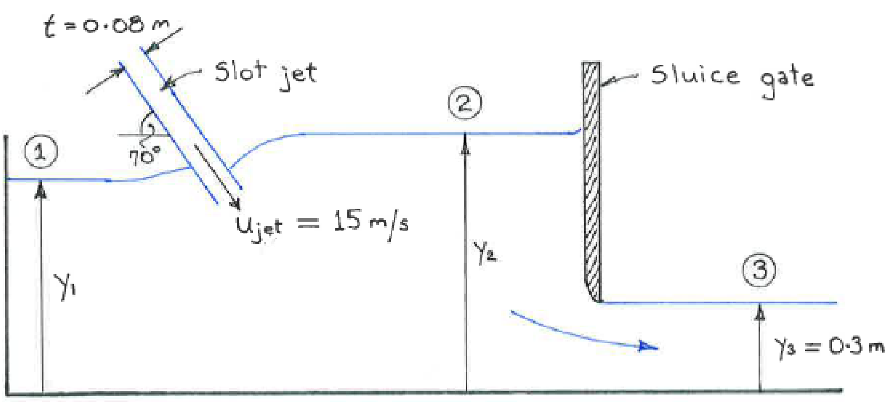

Figure 4: Slot jet entering an open channel, both with width B (into the page)

Data

- Channel width and slot jet width $B = 0.2\,m$
- Thickness of the slot jet $t = 0.08\,m$
- Downwards inclination of the slot jet with respect to the horizontal $\theta = 70\,°$
- Flow velocity of the water in the slot jet just before entering the water body in the channel $u_{\text{jet}} = 15\,m/s$
- Flow depths at the three sections are: y_1, y_2 and $y_3 = 0.3\,m$.

It is reasonable to assume negligible energy losses between Section ② and Section ③ because
(i) the flow in this region does not have any regions of intense turbulence,
(ii) the distance between the 2 sections is relatively small implying friction losses are small and negligible, and
(iii) the flow contraction under the sluice gate is streamlined.

Using the appropriate principles of fluid flow (mass, energy, momentum), complete the following:

1. Find the total head H_3 at Section ③
2. Determine the depth of flow y_2 at Section ②
3. Find the depth of the water at the closed left hand end of the channel y_1.

(**Hint:** You will need to solve a cubic equation at one stage of your calculations, and then select the correct root of the cubic. This type of calculation not infrequently occurs in open channel flow problems.)

6 Question 5: Elements of a Hydro-Power Scheme

Although highly simplified, Figure 5 depicts the main components of a hydro-power scheme. Water stored in a large elevated lake (elevation H_1 above the datum) flows down a pipe (diameter D) capable of withstanding very high pressures, around a bend, through a nozzle (end diameter d) after which it is discharged as a jet into the atmosphere and impacts one of the cups (moving with a linear velocity u_{cup}) of a turbine. Sections ① to ④ are indicated in Figure 5. The velocities (u_2, u_3, u_4) at various locations in the hydro-power scheme are indicated in Figure 5. This diagram will serve the basis for the questions which follow.

Note that to remove some of the dependency of the calculations for one set of problems upon another, 4 data sets are provided. Energy losses due to (i) friction in the pipe and over the turbine cup, (ii) the pipe bend and (iii) the contraction of the flow through nozzle can all be neglected throughout this question.

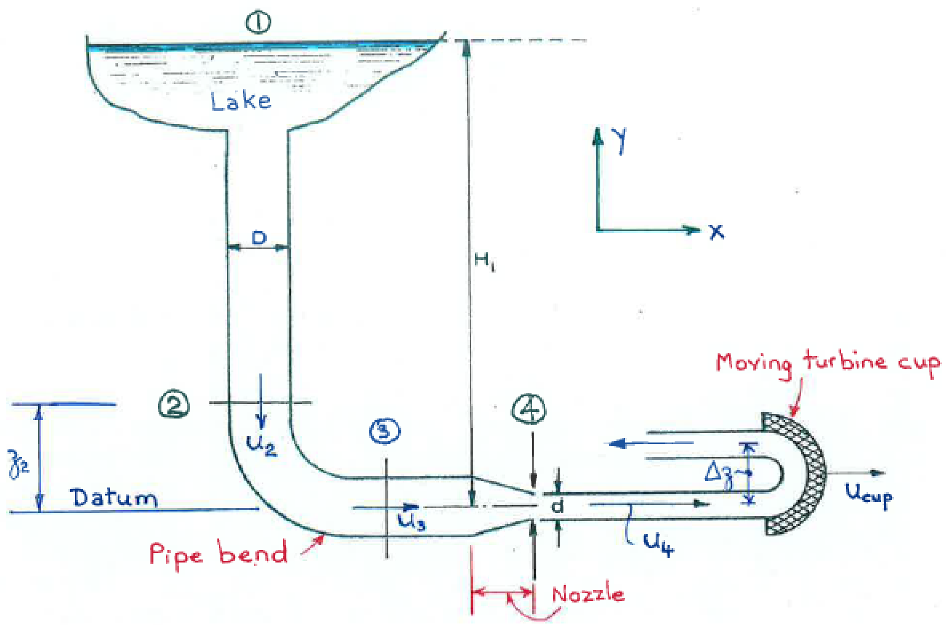

Figure 5: Main elements of a hydro-power scheme.

Data Set 1
$D = 1\,m$, $d = 0.2\,m$, $H_1 = 400\,m$.
(a) Find the flow velocities u_3, u_4.

Data Set 2
$D = 1\,m$, $d = 0.2\,m$, $z_2 = 2.0\,m$, $u_2 = 2.3\,m/s$, $\forall = 2.5\,m^3$ = internal volume of the large diameter pipe between Sections ② and ③. See Figure 6.

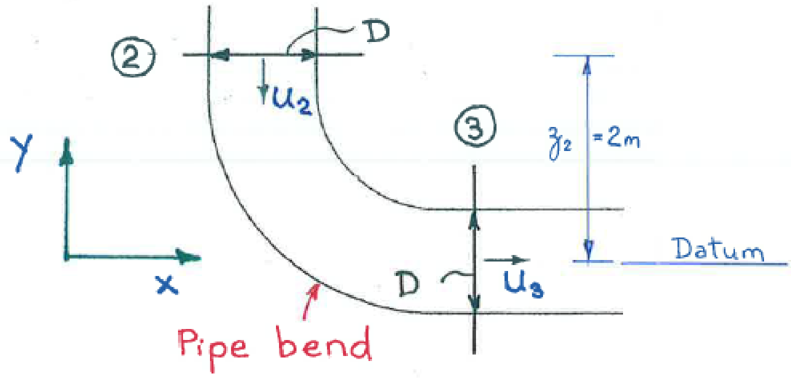

Figure 6: Large diameter pipe bend. The datum coincides with the horizontal pipe axis.

(b) Find the horizontal and vertical forces (F_x, F_y) exerted by the water on the pipe bend. Indicate their directions in a diagram summarising your results.

Data Set 3
$D = 1\,m$, $d = 0.2\,m$, $u_3 = 2.3\,m/s$. See Figure 7.

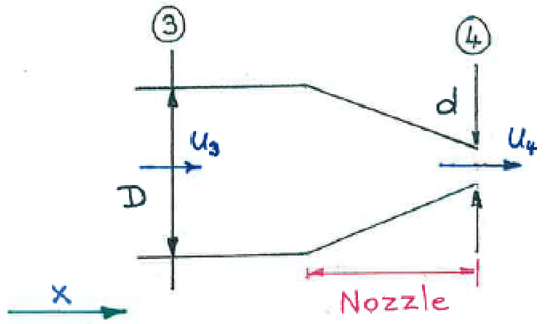

Figure 7: Nozzle at the end of the pipe.

(c) Find the horizontal force (F_n) exerted by the water on the nozzle. Indicate the direction of this force in a diagram.

Data Set 4
$D = 1\,m$, $d = 0.2\,m$, $H_1 = 400\,m$.

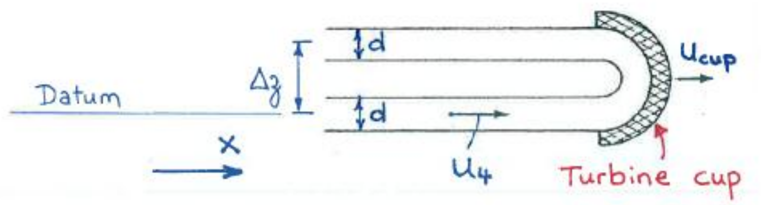

Figure 8: Turbine cup moving to the right at velocity $u_{cup} = (u_4/3)$.

(d) Find the force exerted by the jet of water on the moving turbine cup. Assume (i) that the linear velocity of the turbine cup is $u_{cup} = \frac{u_4}{3}$ to the right and (ii) the elevation difference between the centreline of the jet of water moving to the right towards the cup, and the left moving jet of water after impacting the cup is negligible i.e. neglect Δz in Figure 8